BUCKINGHAM'S PI THEOREM

The dimensions in the previous examples are analysed using Rayleigh's Method. Alternatively, the relationship between the variables can be obtained through a method called Buckingham's π.

Buckingham ' s Pi theorem states that:

If there are n variables in a problem and these variables contain m primary dimensions (for example* M, L, T) *the equation relating all the variables will have (n-m) dimensionless groups.

Buckingham referred to these groups as π groups.

The final equation obtained is in the form of :

$$\pi_1 = f(\pi_2, \pi_3, \ldots \pi_{n-m})$$

The π groups must be independent of each other and no one group should be formed by multiplying together powers of other groups.

This method offers the advantage of being more simple than the method of solving simultaneous equations for obtaining the values of the indices (the exponent values of the variables).

In this method of solving the equation, there are 2 conditions:

a. Each of the fundamental dimensions must appear in at least one of the m variables

b. It must not be possible to fom1 a dimensionless group from one of the variables within a recurring set. A recurring set is a group of variables forming a dimensionless group.

Example: *Question* 5

The relationship of the effect on pressure drop (ΔP) of the variables d, L, p, μ and v

(Question 3).

Answer :

$$f(\Delta P, d, L, p, \mu, v) = O$$

No. of variables = n = 6 (That is: ΔP, d, L, p,μ, v)

No. of fundamental dimensions = m = 3 (That is, [M], [L], [T])

By Buckingham's theorem,

No. of dimensionless groups = n -m = 6-3 = 3

The recurring set must contain three variables that cannot themselves be formed into a dimensionless group. In this case there are two restrictions:

a. Both L and d cannot be chosen as they can be formed into a dimensionless group, (L /d).

b. ΔP, p, and v cannot be used since $(\Delta P / pv^2)$ is dimensionless (we have seen this in the previous example Question 3).

The variables d, v and p are chosen as the recurring set.

The dimensions of these variables are :

$$d = [1]$$

$$v = [LT^{-1}]$$

$$p = [ML^{-3}]$$

Rewriting the dimensions in terms of the variables chosen:

$[L] = d$

$[M] = pd^3$

$[T] = dv^{-1}$

The dimensionless groups are formed by taking each of the remaining variables ΔP, L and μ in turn:

ΔP has dimensions of $[ML^{-1}T^{-2}]$

Therefore $\Delta P\ M^{-1}L\ T^2$ is dimensionless

Substituting the dimensions in terms of variables

$$\pi_1 = \Delta P\ (pd^3)^{-1}\ (d)\ (dv^{-1})^2$$

$$= \Delta P/pv^2$$

L has dimensions of $[L]$

$L[L]^{-1}$ is therefore dimensionless

$\pi_2 = L/d$

μ has dimensions of $[ML^{-1}L^{-1}]$

$\mu[M^{-1}LT]$ is therefore dimensionless

$$\pi 3 = \mu\ (pd^3)^{-1}\ (d)\ (dv^{-1})^2$$

$$= \mu/dvp$$

Thus,

$$f(\Delta P/pv^2,\ L/d,\ \mu/vpd)$$

$$\Delta P/pv^2 = f(L/d,\ vpd/\mu)$$

Example: *Question* 6

The power required by an agitator in a tank is a function of the following variables:

a. Diameter of the agitator

b. Number of rotations of the impeller per unit time

c. Viscosity of liquid

d. Density of liquid

(I) From dimensional analysis using Buckingham's method, obtain a relation between power and the four variables.

(ii) The power consumption is found experimentally to be proportional to the square of the speed of rotation. By what factor would the power be expected to increase if the impeller diameter was doubled?

Answer:

Part 1

$$f(P, D, N, p, \mu) = O$$

Number of variables = 5

Number of dimensions = 3 (i.e. M, L, T)

Number of dimensionless groups = 5-3 = 2

We need to choose the variables so as to represent the dimensions, and hence choose N, D and p

Variable	Dimensions
N	$[T^{-1}]$
D	$[L]$
p	$[ML^{-3}]$

In terms of dimensions,

$N = [T^{-1}]$

therefore, $[T] = N^{-1}$

Similarly,

$D = [L]$,

$[L] = D$

$p = [ML^{-3}]$,

$[M] = p[L^3]$

For the other variables :

The dimensions of power, P, is $[ML^{2}T^{-3}]$

Therefore, $PM^{-1}L^{-2}T^{3}$ must be dimensionless

$$\pi_1 = P\,(pD^3)^{-1}(D)^{-2}(N^{-1})^{3}$$

$$\pi_1 = P / pD^3D^2N^3 = P / pD^5N^3$$

The dimensions of μ is $[ML^{-1}T^{-1}]$

Therefore, $\mu M^{-1}LT$ must be dimensionless

$$\pi_2 = \mu\,(p\,D^3)^{-1}\,(D)\,(N^{-1})$$

$$= \mu\,(p^{-1}D^{-3})\,(D)\,(N^{-1})$$

$$\pi 2 = \mu / pD^2N$$

Hence the relationship derived is,

$$P/ pD^5N^3 = k\,(\mu / pD^2N)^n$$

Where n is any number which is a constant.

Part 2

Experimentally, it is shown that $P \infty N^2$

From the relationship,

$$p = k\,(\mathrm{p}D^5N^3)\,(\mu / D^2N\mathrm{p})^n$$

From this relationship it can be seen that $P \infty N^3 N^{-n}$

or $P \infty N^{3-n}$

But from the experiment, it was found that $P \infty N^2$

Equating the experimental and theoretical findings,

$$3-n=2$$

Therefore $n = 1$

$$P= k\,(D^5N^3\mathrm{p})\,(\mu / D^2N\mathrm{p}) = k\,(D^3N^2\mu)$$

$$p = k\,(D^3N^2\mu)$$

The power required ,

$$P_l = k(D_I{}^3 N_I{}^2 \mu_1)$$

In the second instance, when the size of agitator is doubled,

$$P_2 = k\,(D_2{}^3 N_2{}^2 \mu_2)$$

In this case the viscosity and speed remains the same, so $\mu_l = \mu_2$ and $N_I = N_2$

$$P_2 / P_1 = (D_2 / D_1)^3$$

Since $D_2 = 2 \times D_1$

$$P_2 = 8P_I$$

This means that the power required will be increased 8 times if the diameter of the agitator is

doubled.

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